Tripoli university
Faculty of engineering
EE department
EE313 tutorial (uniform plane waves)

Notes:-

* Perfect dielectric $\Rightarrow \alpha=0, \beta=\omega\sqrt{\mu\epsilon}, \eta=\sqrt{\frac{\mu}{\epsilon}}$. $(\sigma=0)$
* Lossy dielectric $\Rightarrow \sigma\neq 0$.
* Nonmagnetic material $\Rightarrow \mu_r=1, \mu=\mu_o$.
* Loss tangent $=\frac{\sigma}{\omega\epsilon}$ or $\frac{\epsilon''}{\epsilon'}$.
* To obtain the field in real time form:-

$$E(z,t) = Re\left\{\hat{E}(z)\, e^{j\omega t}\right\}$$

Problem #1.

An electromagnetic wave in free space has a wave-length of 0.2m. When this same wave enters a perfect dielectric, the wavelength changes to 0.09m. Assuming that $\mu_r=1$, determine ϵ_r.

Solution

In free space: $\beta_o=\omega\sqrt{\mu_o\epsilon_o} \longrightarrow (1)$

In the perfect dielectric: $\beta=\omega\sqrt{\mu\epsilon}=\omega\sqrt{\mu_o\epsilon_r\epsilon_o} \longrightarrow (2)$

wavelength in free space $\lambda = \frac{2\pi}{\beta_o}$ $\longrightarrow (3)$

wavelength in the dielectric $\lambda = \frac{2\pi}{\beta}$ $\longrightarrow (4)$

Dividing equation (4) by equation (3):

$$\frac{0.09}{0.2} = \frac{\frac{2\pi}{\beta}}{\frac{2\pi}{\beta_o}} = \frac{\beta_o}{\beta} = \frac{\omega\sqrt{\mu_o \epsilon_o}}{\omega\sqrt{\mu_o \epsilon_r \epsilon_o}} = \frac{1}{\sqrt{\epsilon_r}}$$

$\therefore \epsilon_r = 4.94$

Problem #2

In some region $\sigma = 0.01$, $\epsilon = 2\epsilon_o$, $\mu = \mu_o$. The magnitude of electric field at $z=0$ is 100 V/m. At frequency of 100MHz, write the instantaneous expression for E and H. At what depth the E field magnitude reduce to 1%.

Solution

$$\frac{\sigma}{\omega\epsilon} = 0.899$$

$$\alpha = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{\frac{1}{2}} = 1.23\ \text{Np/m.}$$

$$\beta = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{\frac{1}{2}} = 3.209\ \text{rad/m.}$$

$$|\hat{\eta}| = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{\frac{1}{4}}} = 229.9\ \Omega$$

$$\angle \hat{\eta} = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right) = 21^\circ$$

$$E^{+}(z,t) = E_m^{+}\, e^{-\alpha z} \cos(\cancel{\beta}\omega t - \beta z)$$

$$= 100\, e^{-1.23z} \cos(2\pi \times 100 \times 10^{6} t - 3.209z)$$

$$H^{+}(z,t) = \frac{E_m^{+}}{|\eta|}\, e^{-\alpha z} \cos(\omega t - \beta z - \angle \eta)$$

$$= 0.435\, e^{-1.23z} \cos(\cancel{3.209}\, 2\pi \times 100 \times 10^{6} t - 3.209z - 21^\circ)$$

The magnitude of the E-field is $E_m^{+} e^{-\alpha z}$. this will reduce to 1% of 100 (or 1) at distance d:-

$$100\, e^{-1.23d} = 1 \Rightarrow d = 3.744 \text{ m.}$$

Problem #3

The electric field of a wave propagating in a nonmagnetic lossy dielectric is $\hat{E}^{+}(z) = \vec{a}_x\, 10 e^{-\gamma z}$, $\gamma = 3.93 + j4.018$ with a frequency of 20MHz. Find the magnetic field of the wave.

Solution:

To find H, we have to find $\hat{\eta}$, but we don't have ϵ and the losstangent $\frac{\epsilon''}{\epsilon'}$.

$$\gamma = \alpha + j\beta = 3.93 + j4.018 \Rightarrow \alpha = 3.93, \ \beta = 4.018$$

$$\frac{\beta}{\alpha} = \frac{4.018}{3.93} = \frac{\frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2}+1\right]^{\frac{1}{2}}}{\frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2}-1\right]^{\frac{1}{2}}}$$

$$\therefore \frac{\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2}+1}{\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2}-1} = 1.0453$$

After some algebric manipulations:

$$\frac{\epsilon''}{\epsilon'} = 44.9$$

By substituting this value into the expression of α:-

$$3.93 = \frac{2\pi \times 20 \times 10^6 \times \sqrt{4\pi \times 10^{-7} \times \epsilon_r \times 8.854 \times 10^{-12}}}{\sqrt{2}}\left[\sqrt{1+(44.9)^2}-1\right]^{\frac{1}{2}}$$

$$3.93 = 1.963\sqrt{\epsilon_r} \Rightarrow \epsilon_r = 4.$$

And now we can find $\hat{\eta}$.

$$|\hat{\eta}| = \frac{\sqrt{\mu/\epsilon}}{\left[1+\left(\frac{\epsilon''}{\epsilon'}\right)^2\right]^{\frac{1}{4}}} = 28.1\ \Omega$$

$$\angle\hat{\eta} = \frac{1}{2}\tan^{-1}\left(\frac{\epsilon''}{\epsilon'}\right) = 44.36^\circ$$

$$\hat{H}_y^+(z) = \frac{\hat{E}_x^+(z)}{\hat{\eta}}$$

$$= 0.3559\, e^{-3.93z}\, e^{-j(4.018z + 44.36^\circ)}$$

Note that the angle of η is nearly 45° because of the large loss tangent.

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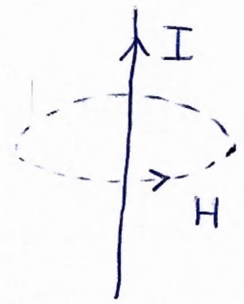
EE313 Tutorial (Magnetic polarization)

Problem (3-26)

Note the similarity between this problem and example (1-17).

a)

Using right hand rule the H field of a stright current carrying conductor is found as in fig(1).

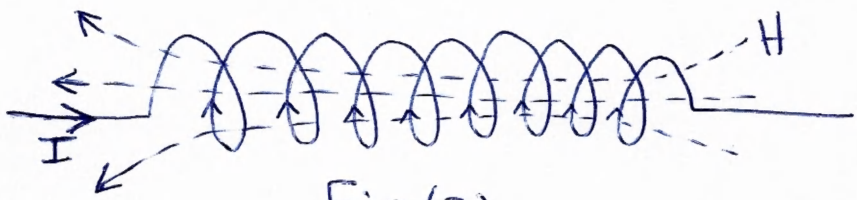


fig(1)

And for single loop by applying the same right hand rule, H will be as shown in fig(2).

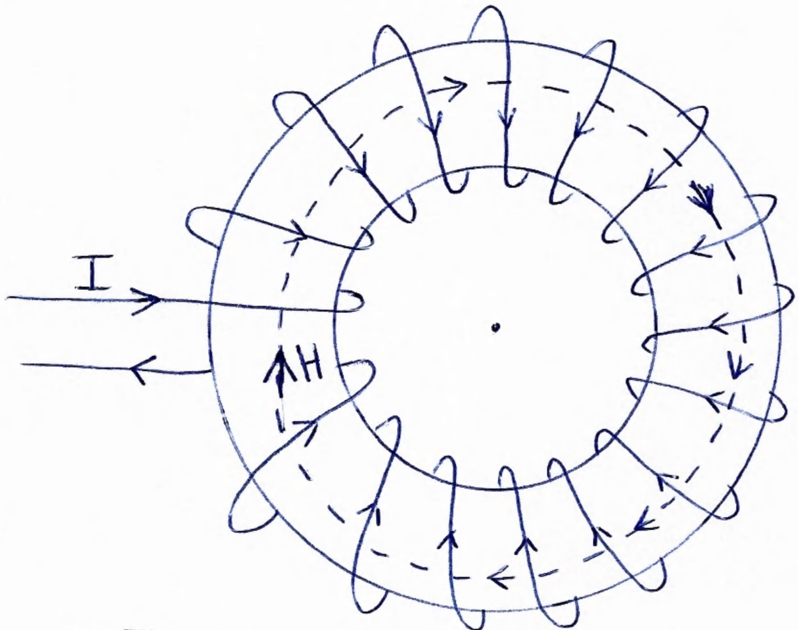
fig(2)

And for n turns solenoidal coil the magnetic field will be as in fig(3)



Fig(3)

And if this solenoid is bent into a toroide we find that the H field inside the toroid must be in $\vec{a}_\phi$ direction as shown in fig(4).



Fig(4), (note that Z-axis is directed out of the paper).

b)

Now, for the toroid of our problem if we take ampere's path for $\rho < a$ as shown in fig(5a) there will be no current crossing the area bounded by the path and hence $H = 0$.

For $a < \rho < c$ (inside the toroid and no difference between the air and the magnetic material) ampers path is shown in fig(5b)

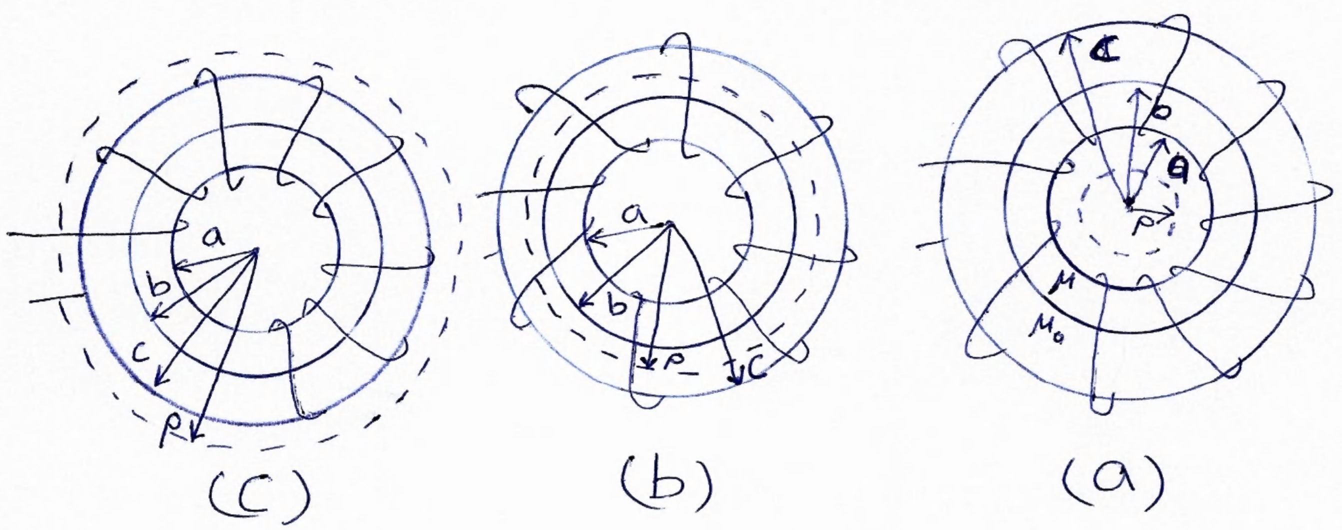


Fig (5)

Here, the current I crosses the area bounded by the path n times. using Ampere's law:-

$$\oint_{l} H_\phi \vec{a_\phi} \cdot dl = nI$$

$$\int_{0}^{2\pi} H_\phi \vec{a_\phi} \cdot \rho d\phi \vec{a_\phi} = nI$$

$$H_\phi = \frac{nI}{2\pi\rho} \quad , a<\rho<c$$

For $\rho > c$, the current I enter the area bounded by the path in opposite directions n times, hence the are cancelling each other in Ampere's law makes $H = 0$. (see fig 5c).

H is the same in the magnetic material and in the air but $B = \mu H$ is not the same since μ for the magnetic material is $\mu_r \mu_o$ and μ for the air is μ_o.

For $a<\rho<b$:

$$\vec{B} = \vec{a}_\phi \frac{\mu_r \mu_o n I}{2\pi\rho}$$

For $b<\rho<c$:-

$$\vec{B} = \vec{a}_\phi \frac{\mu_o n I}{2\pi\rho}$$

c)

$$\vec{M} = \frac{\vec{B}}{\mu_o} - \vec{H}.$$

For $a<\rho<b$:

$$\vec{M} = \left[\frac{\mu_r n I}{2\pi\rho} - \frac{nI}{2\pi\rho}\right]\vec{a}_\phi = \vec{a}_\phi \frac{nI(\mu_r - 1)}{2\pi\rho}$$

For $b<\rho<c$:

$$\vec{M} = \frac{nI}{2\pi\rho} - \frac{nI}{2\pi\rho} = 0.$$

d) For $a<\rho<b$:-

$$\vec{J}_m = \vec{\nabla} \times \vec{M} = \begin{vmatrix} \frac{\vec{a}_\rho}{\rho} & \vec{a}_\phi & \frac{\vec{a}_z}{\rho} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & \frac{nI(\mu_r - 1)}{2\pi} & 0 \end{vmatrix} = 0$$

And now, we want to find J_{sm} (surface current density on the surface of the magnetic material due to polarization).

$$\vec{J}_{sm} = -\vec{n} \times \vec{M}$$, $\vec{n}$ is the normal vector on the surface.

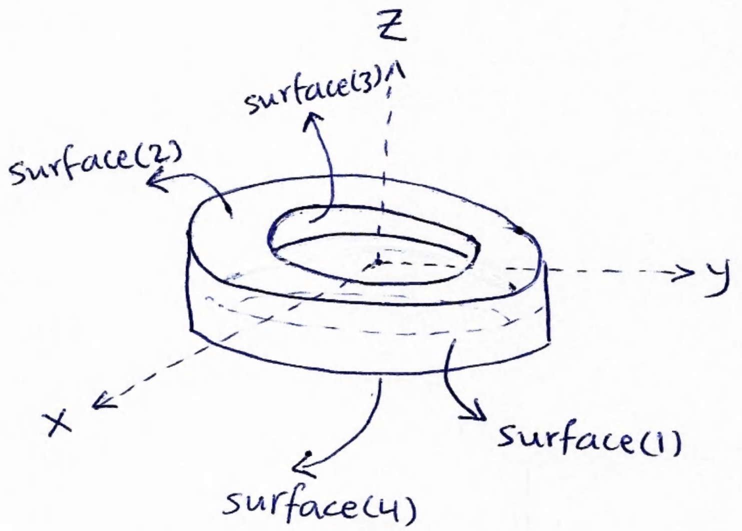


Fig (6)

For surface (1) $(\vec{n} = \vec{a}_\rho)$:

$$\vec{J}_{sm} = -\vec{a}_\rho \times \vec{a}_\phi \frac{nI(\mu_r-1)}{2\pi\rho} = -\vec{a}_z \frac{nI(\mu_r-1)}{2\pi\rho}$$, n is number of turns

For surface(2) $(\vec{n} = \vec{a}_z)$:

$$\vec{J}_{sm} = -\vec{a}_z \times \vec{a}_\phi \frac{nI(\mu_r-1)}{2\pi\rho} = \vec{a}_\rho \frac{nI(\mu_r-1)}{2\pi\rho}$$

For surface (3) $(\vec{n} = -\vec{a}_\rho)$:

$$\vec{J}_{sm} = \vec{a}_\rho \times \vec{a}_\phi \frac{nI(\mu_r-1)}{2\pi\rho} = \vec{a}_z \frac{nI(\mu_r-1)}{2\pi\rho}$$

For surface (4) ($\vec{n} = -\vec{a_z}$)

$$\vec{J}_{sm} = +\vec{a_z} \times \vec{a_\phi} \frac{nI(\mu_r - 1)}{2\pi\rho} = -\vec{a_\rho} \frac{nI(\mu_r - 1)}{2\pi\rho}$$

Fig (7) shows a sketch of Jsm on the magnetic material :-

note that for surface (1) ($\rho = b$) and for surface (3) ($\rho = a$).

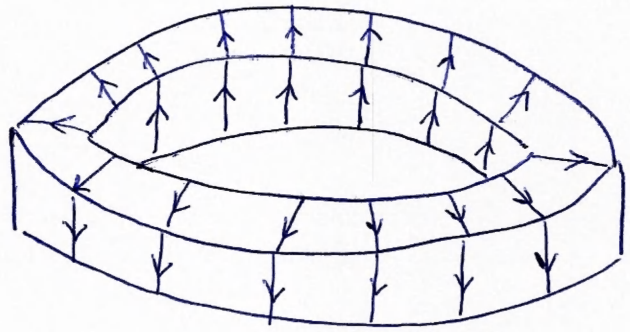


Fig (7)

Problem (3-27)

The plane $z=0$ separates the space into two regions, $z>0$ is air and $z<0$ is a magnetic material with $\mu_r = 4$.

$\vec{B}_1 = 0.3\vec{a_x} + 0.4\vec{a_y} + 0.5\vec{a_z}$ in air is sketched in fig(1). note that the normal vector on this surface is $\vec{a_z}$.

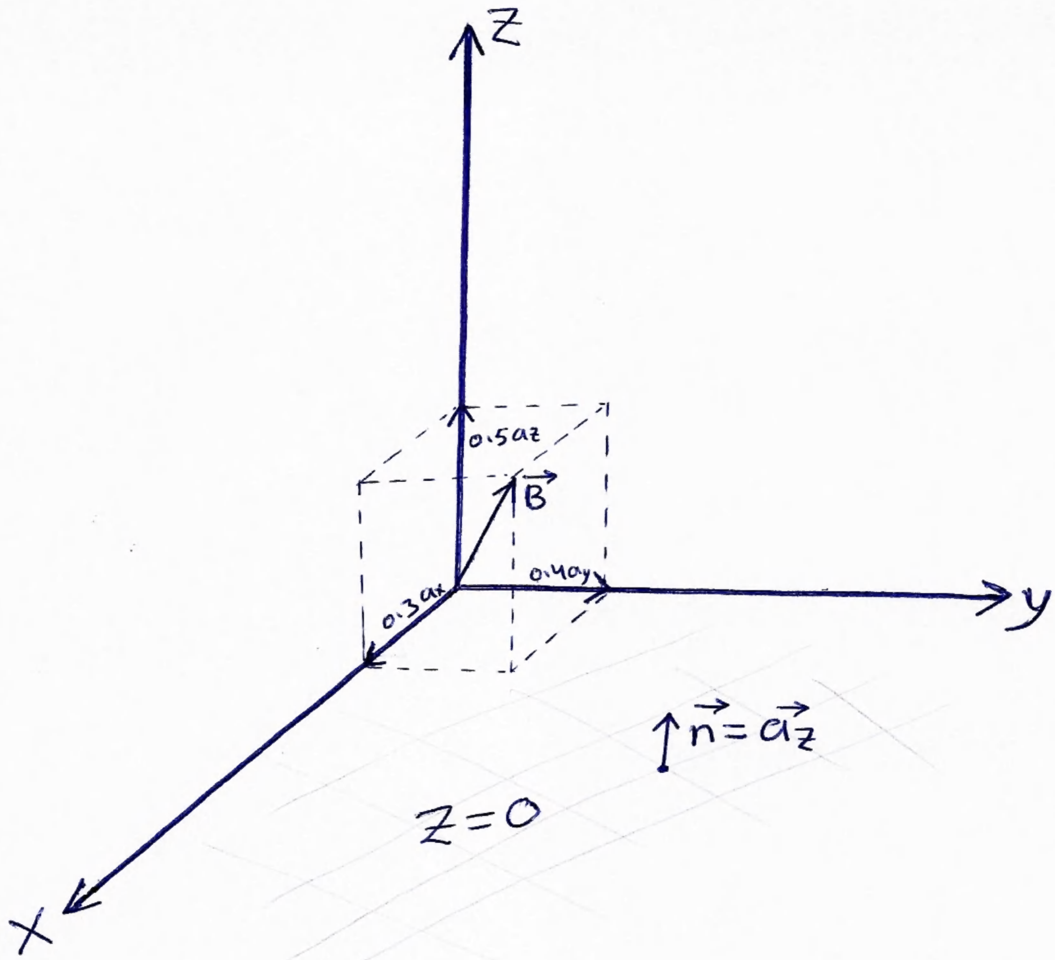


Fig (1)

$$\vec{H}_1 = \frac{\vec{B}_i}{\mu_0} = \frac{0.3}{\mu_0}\vec{a_x} + \frac{0.4}{\mu_0}\vec{a_y} + \frac{0.5}{\mu_0}\vec{a_z}$$

From boundary conditions:

$B_{n1} = B_{n2}$ (normal components of B in each region are equal).

$\therefore B_{1z} = B_{2z} = 0.5\vec{a_z}$

$H_{t1} = H_{t2}$ (tangential components of H are equal).

$\therefore \frac{0.3}{\mu_0}\vec{a_x} + \frac{0.4}{\mu_0}\vec{a_y} = H_{t2}$

$B_{t2} = \mu_r \mu_0 H_{t2} = 1.2\vec{a_x} + 1.6\vec{a_y}$

$\therefore \vec{B_2} = 1.2\vec{a_x} + 1.6\vec{a_y} + 0.5\vec{a_z}$.

The angle between $\vec{B_2}$ and the normal on the surface θ_2 is equal to :-

$$\cos\theta_2 = \frac{\vec{a_z} \cdot \vec{B_2}}{|\vec{B_2}|} = \frac{0.5}{\sqrt{1.2^2+1.6^2+0.5^2}} \Rightarrow \theta_2 = 76^\circ.$$

Similarly $\theta_1 = 45^\circ$.

Problem (3-29)

$$\vec{E_1} = \underbrace{-15\vec{a_x} + 20\vec{a_y}}_{E_{t1}} + 30\vec{a_z}$$

$$\vec{D_1} = \epsilon_0 \vec{E_1} = -15\epsilon_0\vec{a_x} + 20\epsilon_0\vec{a_y} + \underbrace{30\epsilon_0\vec{a_z}}_{D_{n1}}$$

From boundary conditions

$$D_{n1} = D_{n2} = 30\epsilon_0\vec{a_z} \Rightarrow E_{n2} = \frac{30\epsilon_0}{4\epsilon_0} = 7.5\vec{a_z}$$

$$E_{t1} = E_{t2} = -15\vec{a_x} + 20\vec{a_y}$$

$$\therefore \vec{E_2} = -15\vec{a_x} + 20\vec{a_y} + 7.5\vec{a_z}$$

$$\cos\theta_2 = \frac{\vec{a_z} \cdot \vec{E_2}}{|\vec{E_2}|} = \frac{7.5}{\sqrt{15^2+20^2+7.5^2}} \Rightarrow \theta_2 = 73.3^\circ$$

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Fall 2012.